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We will use a proof by induction based on the number of vertices in the graph. If the graph has five or fewer vertices, then five colors are enough to color it.

For the inductive step, we consider a vertex labeled as X in the graph. According to Theorem 4.8, this vertex X has a degree of 5 or less. We remove this vertex X from the graph, resulting in a new graph called G'. By the induction hypothesis, we know that G' can be colored using five colors.

Now, in the original graph G, we examine the degree of the vertex X. If the degree of X is less than 5 or if its neighboring vertices (labeled as X1, X2, X3, X4, and X5) are already colored using four colors or fewer, we can assign the fifth color to vertex X.

However, if X1, X2, X3, X4, and X5 are already colored with different colors in the graph G', as shown in Figure 4.10 (top), then we can color vertex X differently from its neighboring vertices.

In summary, by using the induction hypothesis and considering the degree and colors of the neighboring vertices, we can prove that the original graph G can be colored using five colors, ensuring that no adjacent vertices share the same color.

Consider a blue vertex labeled as 1 and a red vertex labeled as 3. If there is no direct path between vertex 1 and vertex 3 that connects a blue vertex to a red vertex, we can swap the colors along the path from vertex 1 to another vertex, let's call it 6, and color vertex 6 as blue. However, if there is a path that connects vertex 1 and vertex 3 with alternating blue and red vertices, we create a closed loop by adding an extra vertex, let's call it X, and connecting it to vertex 1 and vertex 3. This loop divides the plane into an inner region and an outer region.

Now, let's consider another situation involving a green vertex labeled as 2 and an orange vertex labeled as 4. Since vertex 2 is inside the loop we created earlier and vertex 4 is outside, there cannot be a direct path connecting them without crossing the loop. This contradicts the assumption that the graph is planar. Therefore, vertex 2 and vertex 4 must be part of two separate unconnected chains of green and orange vertices.

To solve this, we can swap the colors along the chain that contains vertex 2. After that, we can color the extra vertex X as green, resulting in a coloring of the graph using five colors.

In summary, by applying these color-swapping techniques and creating additional loops, we can achieve a five-coloring of the graph, ensuring that no adjacent vertices share the same color.

